PROBLEM 1

1) We want the equation of a line, so we need
- a point
- a vector (giving the direction)

The point is the origin $(0,0)$
Since the line must be parallel to another line, we just need to compute the direction vector of the other line.
But the other line is the tangent to $\begin{aligned} x &= t^2+t \\ y &= t^3-1 \end{aligned}$ at $(2,0)$

so we only need to compute the tg vector to the curve at the point $(2,0)$.

First we find for which value(s) of t the curve is at $(2,0)$

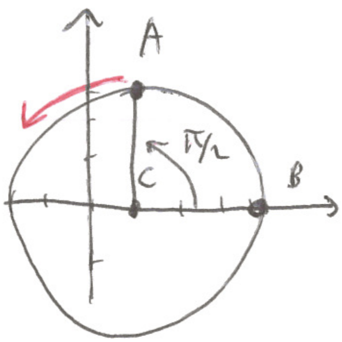
$$t^2+t=2$$
$$t^3-1=0 \quad \rightarrow \quad t=1$$

The second equation gives $t=1$. The first equation has 2 solutions, but $t=1$ is the only solution of the second equation, so we don't care about the other solution.
$t=1$ solves the first eq. so $\boxed{t=1}$ is the only solution of the system $\begin{cases} t^2+t=2 \\ t^3-1=0 \end{cases}$.

Now the tg vector is $\vec{v} = \langle 2t+1, 3t^2 \rangle$ evaluated at $t=1$ so $\vec{v} = \langle 3, 3 \rangle$

So the line we want is $\boxed{\begin{cases} x = 3t \\ y = 3t \end{cases}}$ or $\boxed{y=x}$.

2)



The cartesian eq. is $(x-1)^2+y^2=9$

Since it is oriented anticlockwise we will use cos for x and sin for y

$$\begin{cases} x = 1 + 3\cos t \\ y = 0 + 3\sin t \end{cases} \quad 0 \le t \le 2\pi$$

this parametrization starts at $x = 1 + 3\cos 0 = 4$, $y = 0 + 3\sin 0 = 0$ $B=(4,0)$

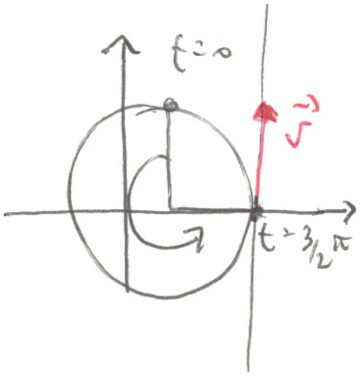
So we need to shift it of $\pi/2$ to start at $(1,3)$

$$\boxed{\begin{cases} x = 1 + 3\cos(t+\pi/2) \\ y = 3\sin(t+\pi/2) \end{cases} \quad 0 \le t \le 2\pi}$$

(You can check that for $t=0$
$x = 1 + 3\cos(\pi/2) = 1$
$y = 3\sin(\pi/2) = 3$)

For the tangent at $(4,0)$ we need to find the corresponding t by solving
$$\begin{cases} 1 + 3\cos(t+\pi/2) = 4 \\ 3\sin(t+\pi/2) = 0 \end{cases}$$

But it is clear from the parametrization that this will occurr at $t = \frac{3}{2}\pi$.



The tg vector is $\vec{v} = \langle -3\sin(t+\pi/2), 3\cos(t+\pi/2) \rangle$

and at $t = \frac{3}{2}\pi$ $\vec{v} = \langle -3\sin(2\pi), 3\cos(2\pi) \rangle$
$= \langle 0, 3 \rangle$

The point is $(4,0)$ so the tg is

$$\begin{matrix} x = 4 + 0 \cdot t \\ y = 0 + 3 \cdot t \end{matrix} \quad \boxed{\begin{cases} x = 4 \\ y = 3t \end{cases}} \quad \text{or} \quad \boxed{x=4}$$

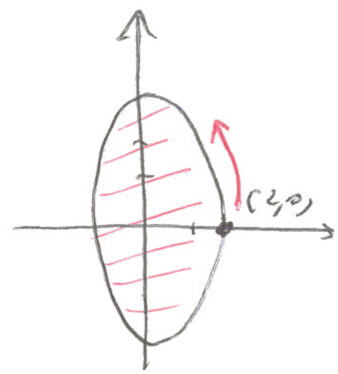
3) the parametric eq. of an ellipse with horiz. semiax. a and vertical semiaxis b, centered at (x_0, y_0) is

$$\begin{cases} x = x_0 + a\cos t \\ y = y_0 + b\sin t \end{cases} \quad 0 \le t \le 2\pi \qquad \text{(anticlockwise)}$$

In our case $(x_0, y_0) = (0,0)$ $\quad a = 2 \quad b = 3$ so

$$\boxed{\begin{cases} x = 2\cos t \\ y = 3\sin t \end{cases} \quad 0 \le t \le 2\pi}$$

For $t = 0$ $\begin{cases} x = 2 \\ y = 0 \end{cases}$ so the starting point is $(2,0)$ and we don't need to shift it.



4) $L = \int_a^b |\vec{v}|\, dt$

$$\vec{v} = \langle 2e^{2t}\sin(2t) + e^{2t}2\cos(2t),\ 2e^{2t}\cos(2t) - e^{2t}2\sin(2t) \rangle$$

$$= \langle 2e^{2t}(\sin(2t) + \cos(2t)),\ 2e^{2t}(\cos(2t) - \sin(2t)) \rangle$$

$$|\vec{v}| = \sqrt{4e^{4t}(\sin(2t) + \cos(2t))^2 + 4e^{4t}(\cos(2t) - \sin(2t))^2}$$

$$= \sqrt{4e^{4t}(\sin^2(2t) + \cos^2(2t) + \cancel{2\sin(2t)\cos(2t)} + \cos^2(2t) + \sin^2(2t) - \cancel{2\sin(2t)\cos(2t)})}$$

$$= \sqrt{8e^{4t}} = 2\sqrt{2}\, e^{2t}$$

$$L = \int_0^\pi 2\sqrt{2}\, e^{2t}\, dt = \sqrt{2}\, e^{2t}\Big]_0^\pi = \boxed{\sqrt{2}\, e^{2\pi} - \sqrt{2}}.$$